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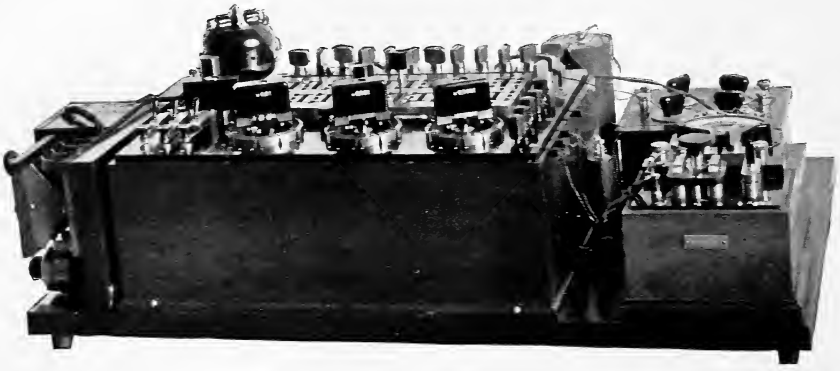

FIG. 1.—*Bridge arranged for oil immersion and automatic temperature control. (Bridge top measures 27 by 39 cm.) View also shows commutator, milliammeter for measuring current through bridge, etc.*

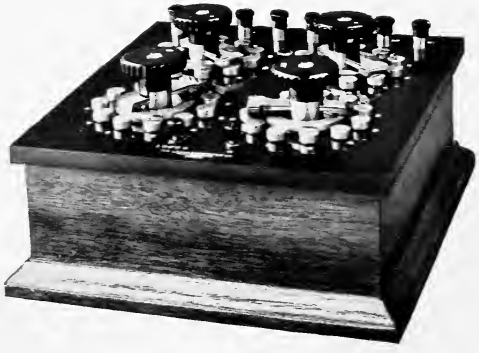

FIG. 2.—*Simple bridge without temperature control. (Bridge top measures 28 by 25 cm.)*

WHEATSTONE BRIDGES AND SOME ACCESSORY APPARATUS FOR RESISTANCE THERMOMETRY

By E. F. Mueller

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1. TYPE OF BRIDGE REQUIRED

A type of Wheatstone bridge which has been found to meet practically all the requirements of precision measurements with resistance thermometers is one having equal ratio arms of rather high resistance (100 ohms or more) and a variable resistance arm of about 100 ohms, variable by steps of 0.0001 ohm. Such a bridge has recently been described.[1] Coils of 0.1 ohm (or even 0.01 ohm) and larger may be connected to mercury contact blocks and short-circuited by suitable amalgamated links. For resistances smaller than 0.1 ohm it is preferable to use a shunting arrangement, as described in the paper referred to [2] or, in some instances, a slide wire may be used. Plug or dial contacts should not, in general, be used directly in series with the variable arm as the variation in such contacts is of the same order of magnitude as the value of the smallest step.

[1] This Bulletin, **11**, p. 571; 1915. [2] Loc. cit., p. 572.

2. IMPROVED SLIDE-WIRE BRIDGE

A consideration of the ordinary bridge diagram, Fig. 3, in which *ab* and *ac* are the equal ratio arms, shows that resistance may be transferred to or from the remaining arms at three points: (1) By moving *d* along *bdc*, which is essentially the plan used in the Callendar-Griffiths bridges [3]; (2) by moving *b* along *dbg;* and (3) by moving *c* along *dcg.* In (2) and (3) the bridge balance is not independent of the resistance of the moving contact as in (1), but the effect of variations of this contact may be reduced to any desired extent by increasing *ab* and *ac* with which the contacts are in series.

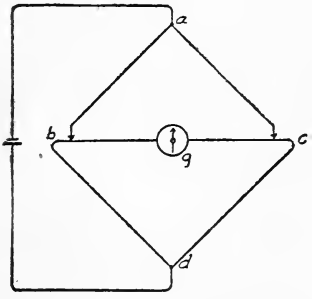

FIG. 3.—*Wheatstone bridge diagram*

The method just described may be used to introduce the 1-ohm decade and the 0.1-ohm decade, a row of 1-ohm coils between contact blocks being connected along *bg* and a row of 0.1-ohm coils along *gc*, the moving points *b* and *c* being either plugs or dial switches.

The above method was described by the writer several years ago to Mr. Leeds, of the Leeds & Northrup Co., and since that time has been used by them in their calorimetric bridges, the steps smaller than 0.1 ohm being secured by the use of a Kohlrausch slide-wire.

3. BRIDGE WITH SHUNT DECADES

Instead of a slide-wire, the smaller steps may also be obtained by the use of shunts. A bridge of this type was designed early in 1913 and has been in use for about two years. The general scheme of connections is shown in Fig. 4. There are two ratio coils of 250 ohms each and these are connected by a short (5 cm) slide-wire, which is used in adjusting them to equality. The ratio coils terminate in long bars which may be connected by means of plugs to the contact blocks of the 1-ohm and 0.1-ohm decades. The 10-ohm coils are connected to binding-post blocks and the desired number of coils are introduced into the circuit by making connection to the appropriate binding post. This method of connection is satisfactory in most instances, but may prove a source of inconvenience when considerable changes in the resistance to be measured are encountered, as may be the case

[3] H. L. Callendar, Phil. Mag. V, 32, p. 104, 1891; E. H. Griffiths, Nature, 53, p. 39, 1895.

when several thermometers are to be measured successively with the same bridge. If this decade is to be much used, it is desirable to make it a plug decade or, perhaps better, to use mercury contacts.

(*a*) *Shunted Decades.*—In choosing the values for the shunted coils the formula for resistance of coils in parallel was put in the following form: If a resistance r is shunted by a resistance R, the effect of the shunt is to reduce r by an amount $\frac{r^2}{R+r}$. In the shunt decades—for example, the 0.01-ohm decade—R is given a series of values such that $\frac{r^2}{R+r}$ successively assumes the values

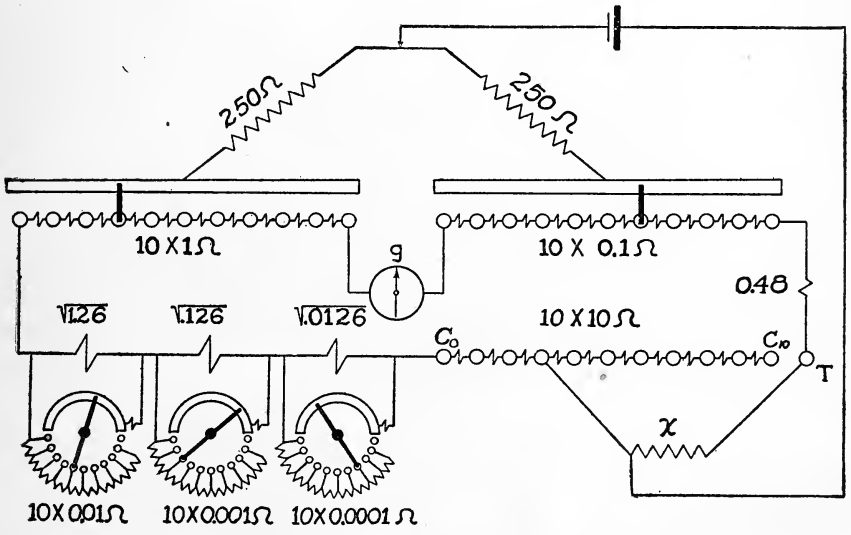

FIG. 4.—*Diagram of connections for bridge with shunt decades*

0.10, 0.09, 0.08—0.00; that is, successive values of R are defined by the equations $R+r=\frac{r^2}{0.10}$, etc., to $R+r=\frac{r^2}{0.00}$. Consequently to secure even-valued coils a value was sought for r^2, which should contain the factors 7 and 9 and therefore be divisible by all numbers below 10. The value $r^2=1.26$ was found to lead to suitable values for the coils of the 0.01-ohm decade. By making r_2 for the 0.001-ohm decade equal to $r/\sqrt{10}$ the successive values of R for this decade are made to differ by a constant amount from the values of R for the 0.01-ohm decade so that the corresponding coils for the two decades, with two exceptions, will be equal. The values of the shunted and shunting coils for the three decades are given in Table 1.

TABLE 1

Resistances of Shunted and Shunting Coils for Last Three Decades

Position of switch	Resistances in ohms			
	0.01-ohm decade—shunted coil, $\sqrt{1.26}=1.1225$		0.001-ohm decade—shunted coil, $\sqrt{0.126}=0.3550$	0.0001-ohm decade—shunted coil, $\sqrt{0.0126}=0.1122$
	Total shunting resistance	Individual coils	Individual coils	Individual coils
0	$12.6-\sqrt{1.26}$	11.478	12.245	12.49
1	$14.0-\sqrt{1.26}$	1.4	1.4	1.4
2	$15.75-\sqrt{1.26}$	1.75	1.75	etc.
3	$18.0-\sqrt{1.26}$	2.25	etc.	
4	$21.0-\sqrt{1.26}$	3.0		
5	$25.2-\sqrt{1.26}$	4.2		
6	$31.5-\sqrt{1.26}$	6.3		
7	$42.0-\sqrt{1.26}$	10.5		
8	$63.0-\sqrt{1.26}$	21.0		
9	$126.-\sqrt{1.26}$	63.0		
10	$\infty-\sqrt{1.26}$	∞		

Instead of the number 1.26, other values may be found more suitable under some circumstances. For example, if it were required that the resistance of the three decades, with switches on zero positions, should be less than 1 ohm, the three shunted coils might be given the values $\sqrt{0.6048}$, $\sqrt{0.06048}$, and $\sqrt{0.006048}$ ($6048=7\times9\times96$).

(*b*) *Adjustment of Ratio Arms.*—The slide-wire between the ratio coils is mounted inside the bridge in a vertical position and the slider is moved by turning a screw which engages the nut on which the slider is mounted. The head of this screw projects through a hole in the bridge top. The bridge is so arranged that the ratio coils may be interchanged at any time, but as this involves manipulating both plugs it is more convenient to adjust the coils to equality as often as may be necessary and not to interchange ratio coils during measurements. The accuracy with which this adjustment can be made is limited only by the sensitivity of the galvanometer used, and errors due to inequality of the ratio coils may therefore be made absolutely negligible.

(*c*) *Temperature Control.*—The bridge is mounted in an oil bath and thermostatic control of the temperature is provided for. The motor for circulating the oil is mounted on the bridge top and drives a screw propeller working in a vertical tube which also contains a heating coil. The oil is circulated downward

through the tube, along the bottom of the box under a false bottom, thence upward and past the coils and through the tube again. A liquid-in-glass thermoregulator is mounted on the lower side of the false bottom. Power for operating the thermostat is supplied from the 110-volt alternating-current line, through a small (40-watt) transformer, which gives voltages up to 16 by steps of 1 volt. This transformer furnishes power for operating the motor and relay and for the heating coil. A switch is so connected that the high voltage can be applied to the heating coil for rapid heating to 30°, at which temperature the thermoregulator is set to operate. A buzzer, also operated from the transformer, indicates when the temperature of 30° has been reached.

A copper coil similar to the sealed coils used for the 10-ohm and 1-ohm decades mounted in the bridge and arranged so that its resistance can be measured with the bridge, shows that in such coils the fluctuations in the temperature of the oil, as the regulator operates, are almost completely damped out.

(*d*) *Effect of Contact Resistances.*—In order to make an estimate of the precision attainable with such a bridge it is necessary to assume the probable variations in the resistances of various kinds of contacts. For this purpose the following figures for the variation of various types of contacts when kept in good condition and correctly manipulated will be used:

Type of contact:	Probable variation
Binding post	0.00002 ohm
Plug	.0001 ohm
Switch	.0002 ohm

The figures quoted can not from the nature of the case be accurate, but they were the best estimates obtainable for the variations in well-made contacts. Granting that a contact of any of the three types will maintain a constant resistance—for example, to 1 microhm, for a short time if not disturbed—values will be computed for the bridge under consideration.

In measuring a resistance of about 25 ohms—for example, in a calorimetric experiment—the effect of any variation of the two plug contacts is only one-tenth of what it would be if these contacts were in the variable arm. Similarly, the maximum effect of variations in the dial switches is only one one-hundredth of the variation of one switch contact. Consequently the total error in the measurement due to contact resistances (the contacts on the binding posts may be left undisturbed) is that due to one or two plugs (reduced 10 times) and one switch (reduced 100 times),

the total amounting to less than 20 microhms, a value comparable with that obtainable with a good mercury contact bridge.

If the resistance to be measured is about 2.5 ohms and is provided with potential terminals and measured in the manner to be described later, the effects of the variations in the plug contacts are reduced 100 times. The total effect of contact resistances in this case will therefore amount to less than 10 microhms, which is as good as can usually be attained with a mercury contact bridge.

The above figures may be summarized in the statement that the precision attainable in the use of the bridge is limited by the contact resistances in series with the ratio coils to about 1 part in 2 000 000 and in the measurement of low resistances is limited by the contact resistances in series with the shunts to about 0.000002 ohm.

The effects of thermoelectromotive forces at the moving contacts are also reduced to such an extent as not to be a source of inconvenience in making measurements.

(*e*) *Performance.*—Tests made with the bridge showed that readings consistent to 1 or 2 microhms could be made on resistances up to 10 ohms, thus indicating that at the time the tests were made the contacts were somewhat better than estimated above. The 1-ohm and larger coils are of the sealed type formerly described[4] and have during two years shown no changes amounting to more than 2 parts in 100 000. The bridge was made by O. Wolff and has proven satisfactory in all respects. A photograph of this bridge forms Fig. 1 (frontispiece).

In this bridge the decade arrangement is used exclusively, and the question may arise as to whether the calibration can be made as accurately as in the bridges in which the 5, 2, 2, 1 combinations are used. There is, of course, an accumulation of errors in each decade, reaching a maximum at the middle of the decade. Such errors could be avoided by comparing groups of coils, but as a matter of fact, in calibration, such high sensitivity may be secured and utilized if the contacts are in sufficiently good condition, that it is sufficient to evaluate each coil separately and add the values. It is worth noting that the decade plan is less subject to error in measuring changes of resistance, as is always done in resistance thermometry. For example, using the 5, 2, 2, 1 arrangement, it is necessary to change all coils if the resistance measured changes from 4 to 6 ohms, while with the decade arrangement it is

[4] This Bulletin, loc. cit., p. 575.

only necessary to add the two coils which are equivalent to the change to be measured.

Successive independent calibrations show that the errors occurring in calibration are not of importance.

The bridge may be calibrated in international ohms without the use of precision auxiliary apparatus (except a standard coil), since by the use of an external variable resistance, each coil of a decade may be compared (by substitution) with the sum of the 10 coils of the next lower decade.[5]

4. DIAL BRIDGE

Another bridge of this general type but improved in details has been designed and constructed. It differs from the one already described principally in employing switches instead of plugs for the 1-ohm and 0.1-ohm decades, and in making the 10-ohm decade usable as a plug decade, or permitting connection directly to the

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[5] For convenience the following account of the usual methods employed in such calibrations is added:

The coils in a decade will be designated as 1, 2, . . . 8, 9, x; so that 9 denotes the sum of the first nine 1-ohm coils and x denotes the sum of the ten 1-ohm coils, and similarly 0.x denotes the sum of the ten 0.1-ohm coils.

The calibration consists of the following steps:

1. Adjust the ratio coils to equality.

2. If a 10-ohm standard is used, measure this standard (preferably by using a commutator) in terms of the x coils, just as a resistance thermometer would be measured. From this a relation $x=10+\triangle$ international ohms is obtained.

3. Connect an external resistance variable by steps of 1 ohm, or preferably, 0.1 ohm to the bridge as for a resistance measurement.

4. Measure a resistance of approximately 1 ohm, first with 1_1, and then with 0.x in the variable arm of the bridge, balance in each case being obtained by using the lower decades of the bridge.

5. Measure a resistance of approximately 2 ohms, first with 1_1+1_2 and then with $1_1+0.x$ in the variable arm, and continue this process up to 10 ohms.

From the observations under (4) and (5) the following equations are obtained:

$$1_1+a_1=0.x+b_1, \text{ etc., to } 1_x+a_x=0.x+b_x.$$

a and b being the readings of the lower decades of the bridge necessary to obtain balance.

Adding the above equations gives

$$x+\Sigma a=10[0.x]+\Sigma b$$

whence

$$0.x=1+0.1[\triangle+\Sigma(a-b)] \text{ international ohms}=1+k.$$

Returning to the original equations we obtain

$$1_1=1+k+b_1-a_1, \text{ etc., to } 1_x=1+k+b_x-a_x.$$

Finally the values so obtained are combined to give

$$1=1_1;\ 2=1_1+1_2, \text{ etc., to } x=1_1+1_2+\ldots\ldots 1_x.$$

A partial check on the arithmetical work is afforded by the fact that the value for x as finally obtained must be equal to $10+\triangle$ ohms.

The calibration of the 0.1-ohm decade proceeds in the same way, the value of 0.x having already been obtained.

It is desirable to carry the observations and computations to one or two decimal places further than the number to be retained, to avoid the effect of the accumulation of errors which occurs in adding the values for the separate coils.

Since resistances variable by sufficiently small steps are not available for calibrating the lower decades, a slide wire may be used in the manner described in this Bulletin (**11**, p. 586, 1915). No elaborate arrar gement is required, a manganin wire about 15 cm long and 1 mm in diameter fastened at the ends and provided with a movable clip for the battery connection being all that is required. The last decade is tested by means of galvanometer deflections.

As the relative accuracy required in the 0.01 ohm and lower decades is not high, calibration of these decades would be necessary only at intervals of several years.

coils through binding posts. The scheme of connections is essentially that of Fig. 4. A plan of this bridge differing very slightly from the one actually constructed is shown in Fig. 5. Mercury contacts could be substituted for the plug contacts without change of arrangement. An arrangement is provided for interchanging the ratio coils which can be used to adjust these coils to equality by means of the slide-wire. This adjustment may be made as follows: By connecting the post marked T' with one of the C' posts—for example, C' 40, and connecting *tc* to C' 20—the first two of the 10-ohm coils may be balanced against the second two. Then on shifting the switches of the 1-ohm and 0.1-ohm decades from the zero positions to R, the ratio coils are interchanged and the bridge balance will be disturbed unless the two are equal. If the coils are not equal, adjustment may be made by means of the slide-wire until no change in balance is noted on interchanging the ratio coils. A special stop with a release ordinarily prevents moving the switchs to the positions R.

The three keys are connected in the battery circuit and so arranged that the key E closes the circuit through an external resistance connected between the posts marked E, the 100 000 key closes the circuit through 100 000 ohms and the *o* key, through no added resistance.

5. BRIDGE WITHOUT TEMPERATURE CONTROL

The bridges so far described are designed for work of very high precision, and thermostatic control of the temperature is practically necessary to secure the accuracy for which the bridges were designed. In a large number of instances, however, a lower degree of precision is permissible. A bridge has recently been designed for measurements of such accuracy as is attainable without thermostatic control of coil temperature. With manganin coils of the average grade, measurements can be made to about 1 part in 25 000 if the coil temperatures are known within 1° or 2°, and the indications of a mercury thermometer with its bulb near the coils should give the coil temperatures within this limit. The bridge, which is designed largely for use with thermometers having a fundamental interval of 1 ohm, has four dial decades, the arrangement of the 1-ohm and 0.1-ohm decades being similar to those in the bridges already described, while the 0.01 and 0.001-ohm decades are secured by the use of shunts. There are also three 10-ohm coils connected to binding posts, so that the bridge

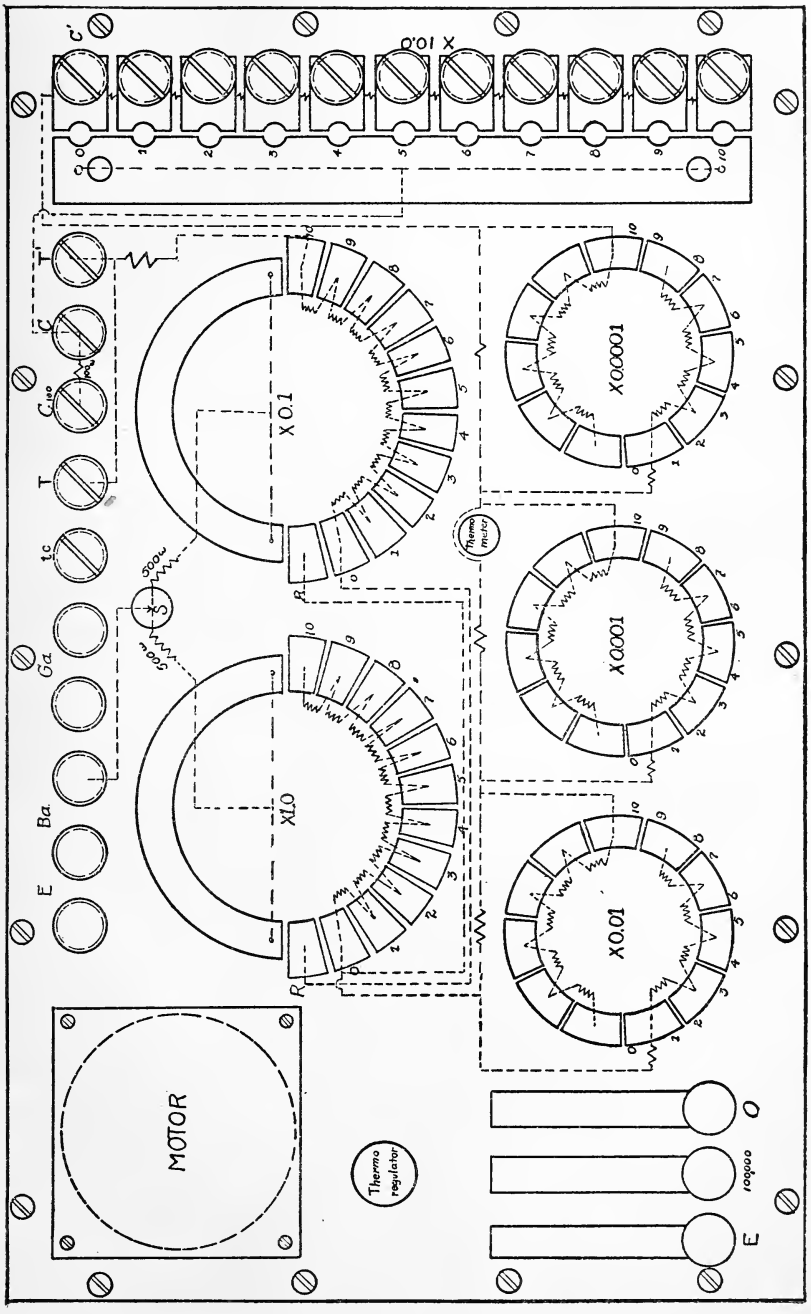

FIG. 5.—*Plan of dial bridge. (Switches not shown. Bridge top measures 24 by 39 cm.)*

has a range up to about 40 ohms, the smallest step being 0.001-ohm, and by utilizing galvanometer deflections readings to 0.0001 ohm may be made.

A bridge of this type has been made for the Bureau by the Leeds & Northrup Co. A photograph of it is shown in Fig 2. It has the advantages of compactness, portability, and simplicity. It will be necessary in some uses of the bridge to take into account the temperature coefficients of the coils in making calculations.

The possibilities of such a bridge may be seen from the following: An accuracy of 1 in 25 000 in resistance measurement corresponds to an accuracy of 0°.002 at −190°, about 0°.01 at room temperatures, about 0°.03 at 500°, and about 0°.05 at 1000°.

By adding another decade giving steps of 0.0001 ohm, this bridge would also be made suitable for calorimetric measurements of high precision, when used with a calorimetric thermometer of 10-ohms fundamental interval. In such measurements the actual temperature of the coils is not of importance, but it is very essential that the temperature remain constant at least to 0.1° during the short time required for an experiment.

A five-dial bridge of this type, which has essentially the arrangement shown in Fig. 4, has been made for the Bureau by the Leeds & Northrup Co.

A resistance thermometer having a fundamental interval of 1 ohm is best adapted for use with a bridge of this type over a wide range of temperature, since the 10-ohm decade is not required, and the resistance is sufficiently high, especially if the potential terminal type is used (see below) to permit readings to the accuracy stated above.

6. MEASUREMENT OF POTENTIAL TERMINAL RESISTANCES

The advantages of thermometers with potential terminals over those which depend upon a compensating method for eliminating lead resistances are too obvious to require discussion. The resistances of such thermometers may be measured with the bridges described in this paper by the method illustrated with reference to Fig. 6*a*. If r_1 and r_2 are the equal ratio arms and the bridge is balanced, the following relation holds

$$R_1 + C = X + T,$$

C and T being the "current" leads. If now T be connected to R and C to r_2 and the battery connected to t, a second balancing gives the relation

$$R_2 + T = X + C.$$

Combining the two equations gives $X = \frac{R_1 + R_2}{2}$.

This method can not be used with a slide-wire bridge of the Callendar-Griffiths type.

In practice the interchange described is effected by the use of a mercury commutator, as shown in Fig. 6*b*. The use of such a commutator involves the introduction of connecting resistances *a* and *b*. It is evident, however, that if two resistances be measured successively, the difference between these resistances will be determined correctly, as the difference of the two values found. One of the two resistances may be equal to zero. The points of

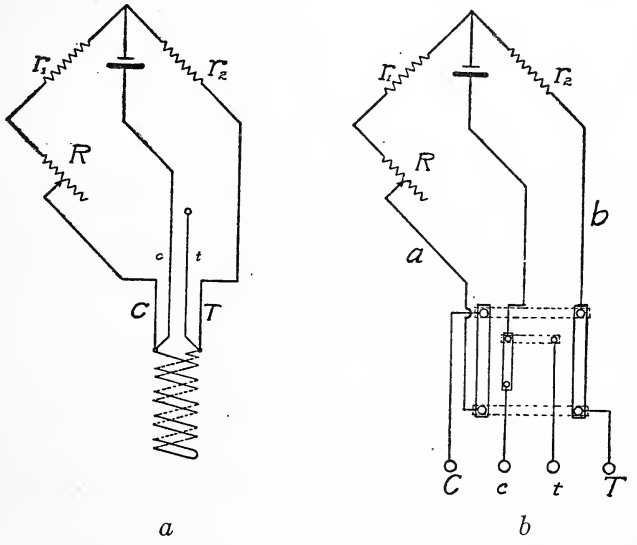

FIG. 6.—*Diagrams to illustrate method of measuring a potential terminal resistance with the Wheatstone bridge*

the commutator to which *C* and *T* are connected are therefore also connected to contact blocks, which may be short-circuited by a plug, the battery connection being also made to the plug at the same time. Since in this case the two potential terminals coincide, the resistance so measured must be equal to zero. Consequently, the difference between the bridge reading obtained in this way (zero balance) and the reading obtained in measuring the resistance of the thermometer will be the resistance of the latter.

It should be noted that the value obtained for the resistance of the thermometer is independent of the connections *a* and *b*, and also independent of the resistance of the plug. It will, however, be affected by variations in the mercury contacts of the commutator.

The method described above was used by H. C. Dickinson and the writer in 1911, and has already been described by F. E. Smith [6] and by D. R. Harper, 3d.[7] A method of measuring the resistance of a four-terminal conductor by a simple bridge method using a commutator had previously been described by Edwards.[8]

In Fig. 7 is shown the plan of a commutator with the necessary binding posts, etc., for making the connections to the bridge and to the thermometer. With the commutator in the normal (*N*) position the connections are those of Fig. 6*a*. For determining

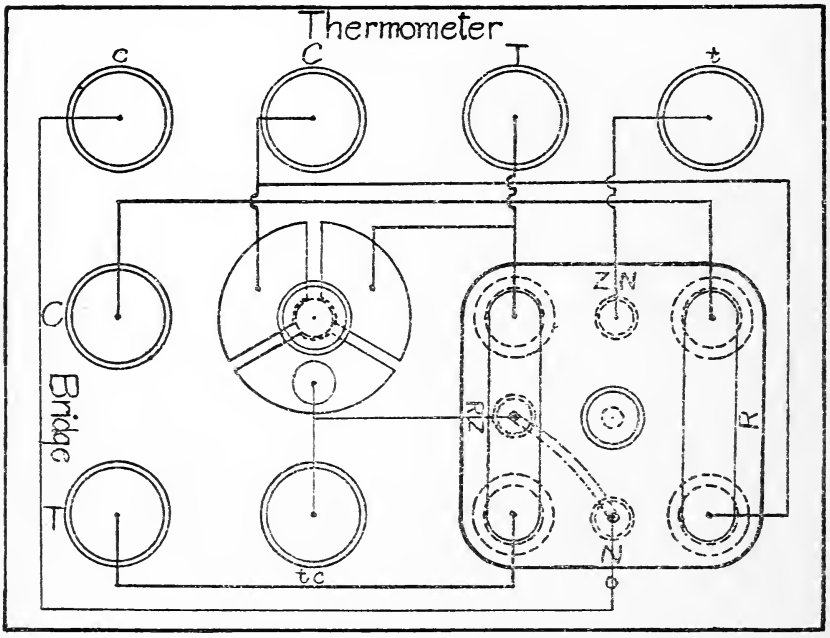

FIG. 7.—*Plan of commutator.* (*Scale full size.*)

the "zero balance," the commutator is used in the positions "*NZ*" and "*RZ*." It is desirable to have the connecting resistances so adjusted that the bridge balances for "*NZ*" and "*RZ*" are practically identical and the same as the balances obtained when a zero resistance is connected to the binding posts to which the thermometer is to be connected.

7. INTERCHANGER FOR CONNECTING SEVERAL THERMOMETERS

In many instances—for example, in intercomparisons of thermometers—it is desirable to measure the resistances of a number of thermometers successively with a single bridge. This may be

[6] Phil. Mag., VI, 24, p. 554; 1912. [7] This Bulletin, 11, p. 296; 1915. [8] Proc. Am. Acad., 40, p. 549; 1905.

done without the necessity of connecting and disconnecting the thermometers by means of the arrangement shown in Fig. 8, which is arranged for four thermometers, of which those not in use are short-circuited by the mercury contact links. A separate switch makes the appropriate battery connections. Such a set of contact blocks may obviously be employed in connection with a commutator as described above. Fig. 9 shows the plan of an interchanger of this kind which is arranged to take four thermometers. The 5-point switch makes the necessary battery connections, the zero position giving the correct connections for the "zero balance." The inside row of binding posts and the switch are mounted above the level of the top, on hard-rubber blocks. The connecting resistances should be adjusted as for the simple commutator described above. With such an arrangement it is possible to measure in

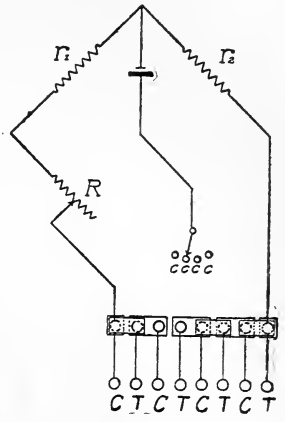

FIG. 8.—*Bridge diagram to illustrate method of connecting several thermometers*

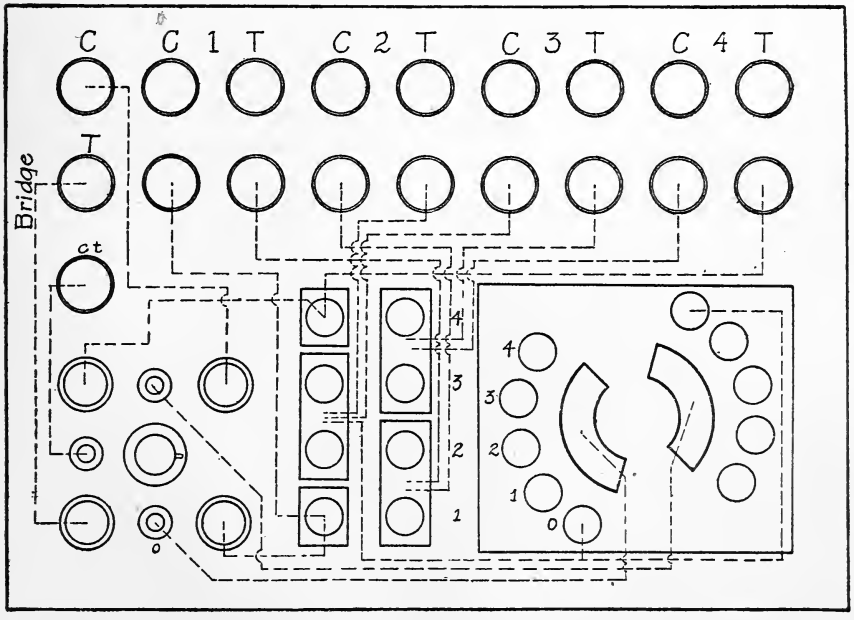

FIG. 9.—*Plan of commutator and interchanger for four thermometers.* (*Scale half size.*)

rapid succession with a single bridge, and without changing any connections, thermometers of the Callendar compensated type, the Siemens type, and the potential-terminal type.

8. NOTATION

The notation used in this paper is applicable to either the Callendar compensated type of thermometers, the Siemens type, or the potential-terminal type. The terminals of the bridge, the commutators, and of the thermometers are marked as shown in Fig. 10.

It is obvious that the potential-terminal thermometer should not be connected directly to the bridge. The Callendar or Siemens type thermometers are connected directly to the bridge, connecting terminals with the same marking. Thermometers of these types may also be connected to the commutator and measured when the commutator is in the normal (N) position. If a thermometer of the Callendar type is connected to the commu-

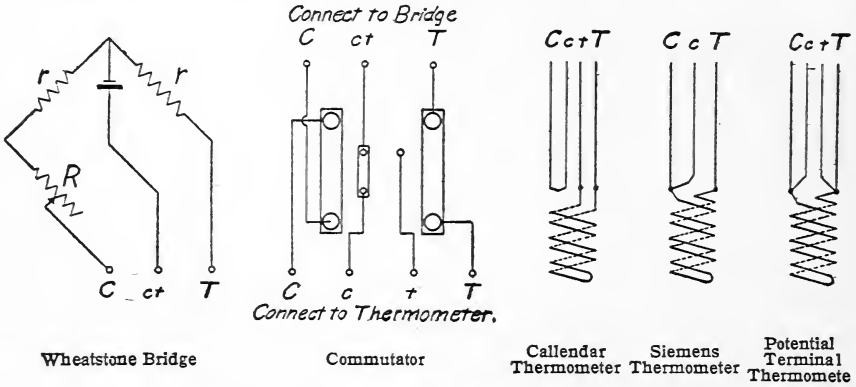

FIG. 10.—*Diagrams to illustrate notation*

tator, both the *c* and *t* leads must be connected to the point *c* of the commutator. It is preferable to mark the two "C" leads of a Callendar thermometer alike and also the two "T" leads. If this is done the marking of the leads will serve to distinguish between a thermometer of the Callendar type and one of the potential terminal type.

A potential terminal thermometer may evidently be used in the Siemens bridge as a three-lead compensated thermometer and eight different combinations, of which four are independent, may be made with the four leads. It is often desirable to be able to use a thermometer either as a three-lead or as a potential terminal resistance. For example, in calorimetric measurements, the use of the commutator would prove inconvenient, and it is improbable that the accuracy attained could be increased in this way. To

avoid ambiguities the resistance of the thermometer when used in the Siemens bridge is defined as the combination

$$X+T-C$$

This requires the use of the lead marked *c* as the third lead. If this thermometer is used with a commutator adjusted as previously described, the data obtained with the commutator in the normal (*N*) position will be applicable to the three-lead thermometer, while the complete data (commutator "*N*" and "*R*") will be applicable to the potential terminal thermometer, and a separate calibration of the three-lead thermometer is not necessary.

9. SUMMARY

A type of Wheatstone bridge suitable for use in resistance thermometry is described, in which plugs or dial switches are used and the circuits so arranged that the errors due to contact resistances are no greater than with the mercury contact bridges heretofore used. The application of these bridges to the measurement of resistances with potential terminals is described. A convenient interchanger by means of which several thermometers may be successively measured with one bridge is also described.

The methods described in this paper have been gradually developed to meet the demands of the Bureau's work in resistance thermometry and represent therefore the joint work of numerous members of the Bureau staff. A number of the arrangements used were suggested by Dr. Dickinson and Dr. Wenner of the Bureau.

WASHINGTON, May 20, 1916.